ENERGY DISSIPATION IN ATMOSPHERIC FLOW

A THESIS

Presented to

The Faculty of the Division of Graduate Studies

by

John M. Stansfield

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in the School of Chemical Engineering

Georgia Institute of Technology

June, 1976

ENERGY DISSIPATION IN

ATMOSPHERIC FLOW

Approved:

R. G. Roper, Chairman

C. G. Justus

M. J. Matteson

Date approved by Chairman: 6/6/76

ACKNOWLEDGMENTS

I would like to express my gratitude to Dr. R. G. Roper for his guidance and support in the preparation of this work. I would also like to thank Dr. C. G. Justus and Dr. M. J. Matteson who served on my Thesis Reading Committee. Their help in the preparation of this thesis and in other areas has been of great value.

A note of thanks is due to Mr. Stanley J. Krivo and to Mr. John C. Yingst. Their friendship was of great assistance to me in undertaking and completing this task.

A very special note of thanks should go to my father for his life-long encouragement to write. I would therefore like to submit this work as a tribute to his memory.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

c	Kolmogrov constant (for structure function)
c_p	specific heat at constant pressure
C_T	thermal structure coefficient
$D_{11}(r)$	structure function along the mean flow
$D_{nn}(r)$	structure function normal to the mean flow
$D_{111}(r)$	third moment
$D_T(r)$	thermal structure function
$E(n)$	energy spectrum
e	turbulent kinetic energy
F	probability distribtuion of the velocity fluctuations
F_0	test statistic
f	frequency
g	gravitational constant
H	vertical heat flux
H_0	null hypothesis
H_1	alternate hypothesis
K	diffusion coefficient
k	wave number
K_c	Kolmogrov constant (for energy spectrum)
K_h	eddy heat diffusivity (conductivity)
K_m	eddy viscosity (diffusivity)
L	characteristic length scale
L_c	correlation coefficient length scale

m	slope
N	Brunt-Vaisala frequency
N_0	number of observations
N_T	rate of destruction of temperature fluctuations
p	pressure
r	distance increment
R_e	Reynolds Number
R_i	Richardson Number
$R(x)$	correlation coefficient, distance
$R(\tau)$	autocorrelation coefficient, time
$S(r)$	skewness of the velocity differences
S_1^2	sample variance of fluctuations along the mean flow
S_n^2	sample variance of fluctuations normal to mean flow
T	temperature
t	time
u	component of the wind along the X axis (Usually assumed parallel to the mean wind direction)
u_0	velocity characteristic of the largest eddy
u_*	friction velocity
$\mathbb{V}$	velocity (a vector)
v	component of the wind along the Y axis
w	vertical component of velocity
x	coordinate along the X axis
Y_d	size of a smoke puff
y	coordinate along the Y axis
z	height coordinate

α	probability of type I error
β	probability of a type II error
Γ	adiabatic lapse rate
ε	rate of energy dissipation
η	Kolmogrov length scale
θ	potential temperature
Λ	ratio of variances
λ	Taylor's microscale
μ	population mean
ν	kinematic viscosity
ρ	density
ρ_0	density in undisturbed atmosphere
σ	Kolmogrov time scale
σ_i^2	population variance
τ	time increment
υ	Kolmogrov velocity scale
χ	concentration

SUMMARY

Atmospheric diffusion is examined from a turbulence point of view. To develop a diffusion model, the statistical theories of G. I. Taylor and A. N. Kolmogrov are presented. Various studies are examined with the objective of ascertaining a suitable estimate of the Kolmogrov constant. The Lilly formula is presented as a basis for calculating the diffusivity, K_m. Radiosonde data is used to calculate a representative profile for the dissipation of energy, ε and the diffusivity, K_m of the atmosphere from the surface upwards to the 20 mb level.

CHAPTER I

INTRODUCTION

> The varying capacity of the atmosphere for transferring and diluting gases, small particles or droplets, is a matter of practical importance in operations or events involving the release or escape of such materials. Problems arising in this field have been among the greatest stimuli to the detailed study of diffusion processes in the atmosphere ...
>
> F. Pasquill, 1962

Much of the early theoretical modeling of atmospheric diffusion is based on a product of the gradient of the diffusing substance and a coefficient of diffusivity. The transport of the substance is assumed to take place along the gradient from regions of high concentration to regions of low concentration. Thus estimates of the new concentration as a result of diffusion require an estimate of the diffusion coefficient.

There have been many field experiments to obtain values for the diffusion coefficient. In addition to the many theoretical problems, there have been many practical problems associated with the search for the diffusion coefficient. Field experiments can be very expensive. Often special efforts such as releasing smoke bombs, specially instrumented aircraft, etc. are required to obtain the diffusion coefficient. A problem with these special efforts is that their cost prevents them from being performed routinely. A solution has to relate the diffusion coefficient to some other parameter that is cheaper to obtain on a routine basis. An example of this is the use of a $\Delta T/\Delta z$ to obtain the spreading coefficients in the Pasquill-Gifford model.

A method is presented in this thesis wherein the diffusion coefficient can be economically estimated directly from data presently collected on a routine basis. This method does not require extensive or elaborate field experiments. The data processing is not complicated. Thus costs may be kept to a minimum. Since the data are collected on a routine basis, the method would permit the calculation on a routine basis also.

The essential concept of the method presented in this thesis is that diffusion is a consequence of turbulence. The premise is advanced that turbulence can be described as an energy process. Specificially, the rate of dissipation of energy present in turbulence may be used to estimate the rate of diffusion taking place in the atmosphere.

CHAPTER II

TURBULENCE THEORY

Introduction

It may not be possible to give a brief or concise definition of turbulence. However, that does not prevent a description of turbulence. Lumley and Panofsky (1964) describe turbulence as:

1. Rotational (and dissipative)
2. Three dimensional
3. Non-linear
4. Stochastic
5. Diffusive
6. Having time and length scales that are large
7. A continuum phenomenon

The diffusive characteristic of turbulence has attracted considerable attention. Tennekes and Lumley (1972) point out the fact that the rate of transfer due to turbulence is several orders of magnitude greater than that due to molecular diffusion. Thus it is postulated that turbulence could be the limiting factor of the diffusion process taking place in a fluid flow. This would suggest that if a defining expression for turbulence could be derived, then perhaps a defining expression for the diffusion process could also be derived.

This is not entirely a new idea. Adolph Fick (1855) presented the hypothesis that has since come to be called K theory. Fick's Law can be stated mathematically in a form of Poisson's equation:

$$\frac{d\chi}{dt} = K \cdot \nabla^2 \chi$$

This states the rate of change with time of some conservative property (χ) is proportional to the Laplacian (∇^2) of that property. This simple expression lends itself to predicting the future state of χ in the atmosphere. V. Bjerknes (1904) stated two necessary and sufficient conditions for solving forecasting problems based upon physical law:

1. A sufficiently accurate knowledge of the state of the atmosphere at the initial time.
2. A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another.

Thus if $\nabla^2 \chi$ is known sufficiently well and if K is known sufficiently well, then the value of χ at some future time may be estimated.

Turbulence Criterion

The stochastic nature of turbulence makes a turbulent flow somewhat difficult to describe. This difficulty was greatly simplified by Osborne Reynolds. In his paper on turbulence, Reynolds (1895) made two important contributions. First, he decomposed the irregular, chaotic motion of turbulent flow into two components:

$$\mathbb{V} = \overline{\mathbb{V}} + \mathbb{V}'$$

where $\mathbb{V}$ = total motion vector

$\overline{\mathbb{V}}$ = constant or mean component

$\mathbb{V}'$ = fluctuating or turbulent component.

Reynolds noticed that under certain conditions the magnitude of $\mathbb{V}'$ was such that the flow could be said to be laminar. For other conditions, the laminar flow became turbulent. This led to the conclusion that at some critical situation a phase change from laminar to turbulent takes place. This led to Reynolds' second major contribution, that of a criterion for turbulence. This criterion today is known as the Reynolds Number:

$$R_e = \frac{WL}{\nu}$$

where R_e = Reynolds Number

W = characteristic velocity

L = characteristic length

ν = kinematic viscosity.

There are two difficulties associated with the use of the Reynolds Number. The first is that of the value of the criterion itself. According to experiments performed by Reynolds, the flow became turbulent for $R_e > 1900$. V. Walfrid Ekman (1911) attempted to duplicate Reynold's results using the very same apparatus at Manchester University that was used by Reynolds. Ekman obtained higher values for R_e than those obtained by Reynolds. The differences can, in part, be explained by two differences in experimental technique. First, Ekman allowed the water used in the experiment to come to rest before starting his experiment. Second, he modified the apparatus to reduce any disturbances generated by the apparatus. This led Ekman to conclude: "The velocity at which turbulence set in was always higher the smaller the initial disturbances; and with sufficient care it came out considerably higher than Reynolds' value".

In addition to Ekman others have explored Reynolds idea for a critical value for turbulence. In examining laminar flow in a boundary layer, W. Tollmien (1929) found a critical value of 420. H. Schlichting (1933) obtained a critical value of 575. The large difference between the results of Reynolds and that of Tollmien and Schlichting can in part be explained by the fact that Reynolds was dealing with flow through a circular pipe while Tollmien and Schlichting were dealing with flow across a flat plate.

The second difficulty with Reynolds Number is one of definition. That is, what criterion is used to define the characteristic velocity and the characteristic length. In a pipe the characteristic length may be easily taken to be the diameter. However, in the atmosphere, no such physical boundaries conveniently exist. Thus comparisons under geometrically similar conditions may be difficult to achieve in actual practice.

Taking a slightly different tack, Lewis F. Richardson examined the atmosphere incorporating a thermodynamic point of view. Richardson (1920) also extended the decomposition concept of Reynolds by decomposing the kinetic energy as well:

$$1/2\rho \ \mathbb{V}^2 = 1/2\rho \ \overline{\mathbb{V}}^2 + 1/2\rho \ \mathbb{V}'^2$$

That is, the kinetic energy may be divided into that associated with the mean and a part associated with the deviation from the mean. Richardson went on to state the turbulent kinetic energy $1/2\rho \ \mathbb{V}'^2$ would increase if

$$\left(\frac{\partial \ \overline{\mathbb{V}}}{\partial \ z} \right)^2 > \frac{g}{\theta} \frac{\partial \theta}{\partial z}$$

where g = gravitational constant

θ = potential temperature

In 1937 Paescke (Richardson, 1952) presented the above as a dimensionless number that is now known as the Richardson Number, R_i:

$$R_i = \frac{\frac{g}{T}\frac{\partial \theta}{\partial z}}{\left(\frac{\partial \backslash V}{\partial z}\right)^2} \simeq \frac{\frac{g}{T}\left(\frac{\Delta T}{\Delta z} + \Gamma\right)}{\left(\frac{\Delta u}{\Delta z}\right)^2 + \left(\frac{\Delta v}{\Delta z}\right)^2}$$

The Richardson Number has the property that if $R_i > 1.0$ then turbulence will subside. If initially $R_i > 1.0$ then no turbulence will develop until $R_i < 1/4$ (Businger, 1968). It should be pointed out that if turbulence has already developed, it may continue to exist up to $R_i = 1.0$ (Reiter, 1969). Having a different critical value depending on initial conditions helps explain the difference in the results obtained by Reynolds and Ekman.

There is a difficulty in using Richardson Numbers. As with Reynolds Number, the length scale (here the height increment) must be carefully defined. A comparison of Richardson Numbers with significantly different height increments could lead to erroneous conclusions.

Turbulence - A Statistical Approach

The concept of Richardson that turbulent fluctuation is an energy process is an important advance. However, the question of scale was not really answered by either Reynolds or Richardson. Another drawback is that essentially only mean values of the flow are all the information that is utilized.

G. I. Taylor, writing in a journal devoted to mathematics, proposed an idea to make greater use of the fluctuating nature of turbulence.

Taylor (1921) introduced the concept of the correlation coefficient to turbulence studies. As a particle moves along in a turbulent flow, its velocity at two different times is related by the Lagrangian velocity correlation coefficient:

$$R(\tau) = \frac{\overline{u(t)\ u(t+\tau)}}{\overline{u^2}}$$

Taylor also introduced the concepts of homogeneous and stationary turbulence. That is, the turbulence is homogeneous if the properties are uniform in space and stationary if uniform in time.

At first Taylor had considered only the Lagrangian case. However, Taylor (1935) also included the Eulerian case as well. This made things convenient as Eulerian measurements are usually more convenient to make. Also in his 1935 paper, Taylor introduced the concept of isotropic turbulence. That is the fluctuations are independent of their axes. He states that if u_1 and u_2 are the velocities measured at two points separated by a distance x then a correlation coefficient R(x) can be defined:

$$R(x) = \frac{\overline{u_1 u_2}}{\overline{u^2}}$$

where $\overline{u_1^2} = \overline{u_2^2} = \overline{u^2}$

This condition that the square of the velocities be equal is the condition of homogeneity introduced by Taylor.

The use of the correlation coefficient enabled a scale to be assigned to the turbulence. After some period of time (or distance) the turbulent fluctuation "forgets" its original value and R(x) becomes zero.

That is, there is a scale L_c such that:

$$L_c = \int_0^\infty R(x)dx$$

In this case x is a distance between two fluctuating points. This of course assumes the integral converges. L_c may be taken as the average "size" of the eddies present without assuming any eddy model.

Taylor (1935) obtained a relation between R(x) at small values of x, i. e. $x/L_c << 1$, and a length scale characteristic of the smallest eddies, the microscale λ.

$$R(x) \simeq 1 - \frac{x^2}{\lambda^2}$$

λ may be taken as the fluctuation length scale dissipating most of the energy (Sutton, 1953).

By analogy in Fickian diffusion the constant K may be expressed as:

$$K = \overline{u'^2} \int_0^t R(\tau)d\tau$$

where t is the time when $R(\tau)$ becomes zero.

In 1938 Taylor (Taylor, 1938) made another important advance. This time he incorporated the idea of a relationship between the correlation coefficient and the spectrum of turbulence. That is, there is a relationship between the autocorrelation $R(\tau)$ and the variations in the various fluctuations observed at a given stationary point. If f is a frequency, then the energy spectrum E(f) can be defined:

$$\int_0^{\infty} E(f)df = 1$$

Taylor, using Fourier analysis, showed that

$$R(\tau) = \int_0^{\infty} E(f) \cos(2\pi f\tau)df$$

and

$$E(f) = 4\int_0^{\infty} R(\tau) \cos(2\pi f\tau)d\tau$$

Three important points can be attributed to the above expressions:

1. Eddy sizes can be thought of as constituting a continuous range of scales.

2. Those eddy sizes contributing most to the kinetic energy can be identified.

3. If $R(\tau)$ is known then $E(f)$ can be found.

In keeping with the idea that turbulence is an energy process, Taylor (1935) proposed that:

> The rate of dissipation of energy of a fluid at any instant depends only on the viscosity, ν, and the instantaneous distribution of velocity. If, therefore, the representation of the essential statistical properties of the velocity field can be expressed by the $R(x)$ curve and similar correlation curves it must be possible to deduce from them the rate of dissipation of energy.

There is a problem. This is described by Lumley and Panofsky (1964). Consider the case where there is a fluctuation about a mean that is not constant. There may be difficulties in separating the scales of the mean value and the scales of the fluctuations. A possible solution might be to take the average over a finite time as the mean. A difficulty could occur if there is a trend in the data. If there is a trend, then the correlation coefficient will not go to zero at large scales.

A. N. Kolmogrov noted that the fluctuations need not be all of the same magnitude. Instead the fluctuations may be described by some probability distribution F. He used this to make two definitions (Kolmogrov, 1941a):

1. The turbulence is locally homogeneous if the distribution F is independent of the position or time.

2. The turbulence is locally isotropic if it is homogeneous and the distribution F is invariant with respect to rotation of the axes.

Kolmogorv's definitions differ from Taylor's in that Kolmogrov requires the F to be steady with time and that restrictions are placed on the velocity differences and not the velocities themselves. This helps ease the problem of a varying mean.

Next Kolmogrov proceeded to define what is now called the "Structure Function". Let u(x) and u(x + r) be two velocity components measured a distance r apart. Then

$$D_{11}(r) = \overline{[u_1(x + r) - u_1(x)]^2}$$

and

$$D_{nn}(r) = \overline{[u_n(y + r) - u_n(y)]^2}$$

where D_{11} is measured along the direction of the mean flow and D_{nn} is measured normal to the mean flow. Taking note of the results of Taylor's 1935 paper and von Karman and Howarth's 1938 paper, Kolmogrov proposed his two Similarity Hypotheses (Kolmogrov, 1941a):

1. The average properties of the small-scale components of turbulence are determined uniquely by the kinematic viscosity ν and the rate of energy dissipation ε.

2. At the large end of the equilibrium range there is an inertial sub-range in which the average properties are determined by the rate of energy dissipation ε only.

The first similarity hypothesis is the basis for three characteristic values for the smallest eddies:

length (microscale): $\eta = (\frac{\nu^3}{\varepsilon})^{1/4}$

time: $\sigma = \frac{\nu}{\varepsilon}$

velocity: $\upsilon = (\nu\varepsilon)^{1/4}$

For the case when r is large in comparison with η, Kolmogrov derived the following:

$$D_{11}(r) \propto \varepsilon^{2/3} r^{2/3}$$

and

$$D_{nn}(r) \propto \frac{4}{3} D_{11}(r)$$

Kolmogrov (1941b) examined the above proportional relationship and using the third moment

$$D_{111}(r) = \overline{[u_1(x + r) - u(x)]^3}$$

noted that for locally isotropic turbulence in an incompressible fluid

$$6\nu\frac{d}{dr} D_{11} - D_{111} = \frac{4}{5}\varepsilon r$$

For small r the third moment is small in comparison to $6\nu \frac{d}{dr} D_{11}$. However, for large r, the size relationship is reversed. Therefore for large r

$$D_{111} \simeq -\frac{4}{5}\varepsilon r$$

Now the skewness S(r) of the velocity fluctuations involves the second and third moments, that is

$$S(r) = D_{111}/D_{11}^{3/2}$$

Now it is possible to write for large r.

$$D_{11} \simeq c\varepsilon^{2/3} r^{2/3}$$

where the Kolmogrov constant is

$$c = \left[-\frac{4}{5(S(r))}\right]^{2/3}$$

The exponent on r of 2/3 may be argued from a dimensional analysis basis. However, the exponent may be estimated from data. Let r_1 and r_2 be two different separation distances over a range of r where the theory holds. Noting that ε is independent of r it is possible to write

$$\frac{D(r_2)}{D(r_1)} = \frac{c\varepsilon r_2^{\,m}}{c\varepsilon r_1^{\,m}} = \left(\frac{r_2}{r_1}\right)^m$$

Taking the log of both sides, it is possible to write

$$m = \frac{\ell n\ D(r_2) - \ell n\ D(r_1)}{\ell n\ r_2 - \ell n\ r_1}$$

This result is the form of the slope of a straight line. Thus when D(r) vs r is plotted on log-log paper, the exponent of r is a straight line over the region where the theory holds. Note also that as long as the increments of r are of equal size, their relative magnitude does not affect the magnitude of m.

Recalling the earlier discussion about correlation coefficients it is possible to express D(r) as a function of R(r)

$$D(r) = 2\overline{u_0^2}\ [1 - R(r)]$$

where $\sqrt{\overline{u_0^2}}$ is the velocity characteristic of the largest eddy.

The Lilly Formula

A sizable amount of the published literature has been devoted to turbulent exchange coefficients, transport coefficients, diffusivities, or austausch coefficients. Two of these coefficients are the eddy heat diffusivity K_h and the eddy viscosity K_m. These two are defined by Lumley and Panofsky (1964) as

$$K_m = -\frac{\overline{u'w'}}{\left|\frac{\partial V}{\partial z}\right|}$$

and

$$K_h = -\frac{\overline{\theta w'}}{\frac{\partial T}{\partial z}}$$

Since turbulence can be anisotropic, both K_m and K_h may acquire vector qualities. Since both momentum and heat may be transported by turbulence with equal effectiveness, it is often assumed that

$$K_m/K_h = 1 \qquad \text{(turbulent Prandtl Number)}$$

D. K. Lilly (Lilly, et al., 1974) has proposed a method by which the eddy heat diffusivity K_h may be estimated using meteorological measurements. The argument made by Lilly is that turbulence in the atmosphere may be characterized by Richardson Numbers near 1/4.

A development will be presented here for K_m. Let the turbulence be characterized by $R_i = 1/4$. That is,

$$1/4 = \frac{\frac{g}{T}\frac{\partial \theta}{\partial z}}{\left(\frac{\partial \mathbb{V}}{\partial z}\right)^2}$$

or

$$\left|\frac{\partial \mathbb{V}}{\partial z}\right| = \frac{4\,\frac{g}{T}\frac{\partial \theta}{\partial z}}{\left|\frac{\partial \mathbb{V}}{\partial z}\right|}$$

Now let N be the Brunt-Vaisala frequency defined by

$$N^2 = \frac{g}{T}\frac{\partial \theta}{\partial z}$$

N is the frequency of the vertical oscillation that a displaced air parcel would have about its equilibrium position. Thus

$$\left|\frac{\partial \mathbb{V}}{\partial z}\right| = 4N^2/\left|\frac{\partial \mathbb{V}}{\partial z}\right|$$

Multiplying both sides of this expression by $-\overline{u'w'}$ yields

$$-\overline{u'w'}\left|\frac{\partial \mathbb{V}}{\partial z}\right| = -4N^2\frac{\overline{u'w'}}{\left|\frac{\partial \mathbb{V}}{\partial z}\right|}$$

Recalling the definition for K_m then

$$-\overline{u'w'}\left|\frac{\partial \mathbb{V}}{\partial z}\right| = 4N^2\ K_m$$

The term $-\overline{u'w'}\left|\frac{\partial \mathbb{V}}{\partial z}\right|$ is the rate of production of turbulent kinetic energy by shear. This rate of production is equal to the sum of the viscous dissipation, ε, and the buoyant removal of energy. If it is assumed that

$$\varepsilon \simeq -3/4\ \overline{u'w'}\left|\frac{\partial \mathbb{V}}{\partial z}\right|$$

with buoyancy effects accounting for the remainder of the production term, then

$$K_m \simeq \frac{\varepsilon}{3N^2}$$

This is the same expression Lilly obtained for heat.

There are some uncertainties in the assumptions relating to the dissipation. Lilly, in his derivation, estimates the buoyancy flux to be 1/3 the dissipation. However, Lenschow (1973) places the buoyancy flux in penetrative convection to be 1/10 the dissipation. If this is the case then

$$K_m \simeq \frac{11}{40}\frac{\varepsilon}{N^2}$$

$$\simeq \frac{\varepsilon}{4N^2}$$

would be approximately correct. Lilly estimates his expression for K_h may be in error up to a factor of 2. Thus it is relative values rather than precise values obtained from either expression that are important.

Lilly, et al., (1974) obtained turbulence data from aircraft measurements in the stratosphere. Using the energy spectrum approach, values for ε were obtained. These ε values were then used to calculate K_h. The heat diffusivity was found to vary with altitude, terrain, and season. Measurements in the summer were higher than those made in the winter. The values for different altitudes and terrain are shown below.

Table 1. Mean Heat Diffusivity Coefficient, $\overline{K}_h$ (Lilly, et al., 1974)

Altitude (km)	Water and Flatlands K_h (m^2/sec)	All Mountains K_h (m^2/sec)
13.7 - 15.2	0.75	0.40
15.2 - 16.8	0.46	0.94
16.8 - 18.3	0.41	0.83
>18.3	0.26	0.81

CHAPTER III

CALCULATIONS MADE BY OTHERS

Calculations of ε by Others

It should be recognized that turbulence is a property of flow. If there is a flow, then there will be kinetic energy associated with that flow. If that flow is turbulent, then it may be possible to speak of a turbulent kinetic energy. Lumley and Panofsky (1964) develop a budget for the turbulent kinetic energy using the energy equation:

$$\frac{\partial e}{\partial t} = u_*^{\,2} \frac{\partial V}{\partial z} + \frac{gH}{c_p \rho T} - \frac{\partial}{\partial z}(\overline{ew}) - \frac{\partial}{\partial z}\left(\frac{\overline{wp}}{\rho_o}\right) - \varepsilon$$

where e = turbulent kinetic energy

$u_*^2 \frac{\partial V}{\partial z}$ = production of mechanical energy

$\frac{gH}{c_p \rho T}$ = production of convective energy

$$\left.\begin{array}{l} \frac{\partial}{\partial z}(\overline{ew}) \\ \frac{\partial}{\partial z}\left(\frac{\overline{wp}}{\rho_o}\right) \end{array}\right\} = \text{vertical flux terms}$$

ε = dissipation term

Typical values of $\frac{\partial e}{\partial t}$ are on the order of a few $\frac{\text{erg}}{\text{gm sec}}$. The mechanical term may have values of $10^3 \frac{\text{erg}}{\text{gm sec}}$ near the surface and decreases rapidly with height. The convective term may be on the same order of magnitude as the mechanical term at heights removed from the surface. R. J. Taylor (1952) examined the case near the ground. Here he measured the flux terms and found them to be small ($<20 \frac{\text{erg}}{\text{cm}^2 \text{ sec}}$). In this situation, assuming $\frac{\partial e}{\partial t} \simeq 0$, the energy equation becomes

$$\varepsilon = u_*^2 \frac{\partial V}{\partial z} + \frac{gH}{c_p \rho T}$$

R. J. Taylor found that under equilibrium conditions ($\frac{gH}{c_p \rho T}$ small) the energy dissipation at 2 meters was:

Time (GMT)	$\varepsilon(\frac{erg}{gm\ sec})$
1145	1150
1202	360
2037	48
2207	8.4

Another technique uses the diffusion of smoke puffs. This method, suggested by Batchelor (1950, 1951), has been used by Gifford (1957). In this method the diameter Y_d of a puff of smoke is measured with time. At first, the puff spreads according to

$$\overline{Y_d^2(t)} - \overline{Y_d^2(0)} \propto t^2$$

At some later time period the size varies according to

$$\overline{Y_d^2(t)} \propto t^3$$

The time when the change from a t^2 law to a t^3 law is

$$t_1 \propto Y_d(0)^{2/3}\, \varepsilon^{-1/3}$$

Thus by measuring t and Y_d it is possible to measure ε.

A third technique utilizes knowledge of the turbulence spectra of the wind. The energy spectrum E(k) of the turbulence may be defined by (one dimension case):

$$\overline{u^2} = \int_0^\infty E(k)dk$$

where k = wave number

From this it is possible to obtain

$$\varepsilon = 15\nu \int_0^\infty kE(k)dk$$

If Kolmogrov's second hypothesis is assumed then

$$E(k) = K_c \varepsilon^{2/3} k^{-5/3}$$

where K_c = a universal constant

Pond, et al., (1963) have developed a technique whereby the constant K_c may be found. The skewness S(r) of the velocity differences observed at two points separated by a distance r is related to the constant K_c by

$$S(r) = -0.100 K_c^{-3/2}$$

Pond, et al., obtained a value of S(r) = -0.32 for r in the inertial range, or K_c = 0.46.

With two hot-wire anemometers located 1 meter above a water surface, Pond et al., measured the following values of ε on July 29, 1962:

Time of Day	ε (cm^2/sec^3)
0140-0210	87
0140-0150	95
0150-0200	90
0200-0210	80

In addition to the value of K_c = 0.46 above, other values for the longitudinal velocity component have been obtained. For the lateral velocity component the appropriate value would be 4/3 K_c. Some values for K_c are:

Experimentor	K_c (longitudinal velocity component)
Batchelor and Townsend (1948)	0.33
Obukhov (1951)	0.40
Grant, et al., (1962)	0.47 ± 0.02
Pond, et al., (1963)	0.47

A fourth method makes use of the "Structure Function" of the Russian school as developed by Kolmogrov (see Chapter II, Turbulence - A Statistical Approach). If Kolmogrov's second hypothesis is assumed, then by a dimensional argument the structure function becomes:

$$D(r) = c\ (r\varepsilon)^{2/3}$$

where c = a universal constant

Gurvich (1961) has examined the relationship of this constant c and the structure function normal to (D_{nn}) and along (D_{11}) the mean flow. He presented the following arrangement:

$$D_{11} = 3/4\ D_{nn} = 3/4\ c\ (\varepsilon r)^{2/3}$$

where $c = 4/3\ [-\frac{4}{5\ S_1(r)}]^{2/3}$

$S_1(r) = D_{111}/D_{11}^{\ 3/2}$ (skewness measured along the mean flow)

Note that a minus sign appears in the expression for c. Several negative values for $S_1(r)$ have been obtained experimentally:

Experimentor	$S_1(r)$ (skewness along flow)	Reynolds Number
Townsend (1948)	-0.38 to -0.25	4.4×10^4
Stewart (1951)	-0.17 to -0.11	4.2×10^4
Obukhov (1951)	-0.8	
Gurvich (1961)	$\begin{cases} -0.45 \pm 0.05 & (r = 25 \text{ cm}) \\ -0.40 \pm 0.06 & (r = 50 \text{ cm}) \end{cases}$	

Gurvich (1961) points out that in locally isotropic flow the skewness measured normally to the flow, $S_n(r)$, should be zero due to symmetry. Any deviation of $S_n(r)$ from zero serves as an indicator for the validity of the theory. In his work he obtained a value of $S_n(r) = 0.03 \pm 0.02$.

Thus several experimental values for c have been obtained. Townsend (1948) used:

$$S(r) = \frac{\overline{(u_2 - u_1)^3}}{[\overline{(u_2 - u_1)^2}]^{3/2}} = -\frac{4}{5} C^{-3/2} \qquad (\text{here } C = \frac{3}{4} c)$$

and obtained a mean value of $c = 4/3(1.53) = 2.04$.

For Gurvich's results using his expression the following mean values are obtained:

$$\begin{aligned} c &= 1.96 \qquad (r = 25 \text{ cm}) \\ &= 2.12 \qquad (r = 50 \text{ cm}) \end{aligned}$$

Lumley and Panofsky (1964) state that the constant K_c of the energy spectrum is related to the structure function constant c by the relationship:

$$4\, K_c \simeq \frac{3}{4}\, c$$

Using this relationship and the values for K_c the following values for c are obtained:

Experimentor	$c\ [D_{11} = \frac{3}{4} D_{nn} = \frac{3}{4} c\ (r\varepsilon)^{2/3}]$
Batchelor & Townsend (1948)	1.76
Townsend (1948)	1.96
Obukhov (1951)	2.13
Gurvich (1961)	1.96
	2.12
Grant, et al., (1962)	2.51
Zubkovski (1962)	2.20
Pond, et al., (1963)	2.45

A modification of the structure function uses fluctuations of the temperature field instead of fluctuations in the velocity field. The structure function takes on the form:

$$D_T(r) = C_T^2 r^{2/3}$$

where $C_T = c\varepsilon^{-1/3} N_T$

$N_T = K_h \left(\frac{\partial\theta}{\partial z}\right)^2$

Vinnichenko and Dutton (1969) report that c = 2.8 in this case. The results of their studies of Clear Air Turbulence (CAT) as determined by Project HICAT are presented in Table 2. The reported CAT is based on the reaction of the aircraft encountering the turbulence.

The dissipation of energy with height has been examimined using many different techniques. Readings and Rayment (1969) used a tethered ballon over the flat open country at Cardington, England. A series of midday ascents up to one kilometer in height were made. During these ascents, measurements of temperature, wind speed, and wind inclination were taken for five minute periods at various heights.

Using the energy spectrum approach:

Table 2. Mean Values of ε and the Thermal Structure Coefficient

z(km)	Reported CAT	$\varepsilon(\mathrm{cm}^2/\mathrm{sec}^3)$	$C_T(°\mathrm{C}^2/\mathrm{cm}^{2/3})$	$N_T(°\mathrm{C}^2/\mathrm{sec})$
18.2	Light	11	3.0×10^{-5}	2.4×10^{-5}
18.3	Light	2.8	3.8×10^{-5}	1.9×10^{-5}
19.0	Light-Moderate	84	1.1×10^{-4}	1.8×10^{-4}
19.3	Moderate	15	2.3×10^{-5}	2.0×10^{-5}
18.3	Moderate	16	4.2×10^{-5}	3.7×10^{-5}
18.3	Moderate	15	5.0×10^{-5}	4.4×10^{-5}
18.9	Moderate	6.1	5.8×10^{-5}	3.8×10^{-5}
18.1	Moderate	11	9.5×10^{-5}	7.5×10^{-5}
18.5	Moderate	480	6.8×10^{-4}	1.9×10^{-3}
18.7	Severe	28	8.2×10^{-5}	8.9×10^{-5}
18.4	Severe	230	6.0×10^{-4}	1.3×10^{-3}
18.7	Severe	190	2.3×10^{-4}	4.8×10^{-4}

Vinnichenko and Dutton also provide approximate classifications for CAT encountered by aircraft on the basis of the energy dissipation ε.

Category of CAT	$\varepsilon(\mathrm{cm}^2/\mathrm{sec}^3)$
None	< 30
Light	30 - 120
Moderate	120 - 1000
Severe	> 1000

$$E(k) = K_c \varepsilon^{2/3} k^{-5/3}$$

where K_c = 0.63 (from Panofsky and Pasquill, 1963).

Readings and Rayment obtained some interesting profiles of ε with height. One of these is shown in Figure 1. In this instance an inversion existed and the rate of dissipation below, through, and above the inversion was determined. The two rates ε_1 and ε_2 are the result of different electrical smoothing circuits used in their recording equipment. The important observation is the effect an inversion has on the energy dissipation. Note that ε decreases with height below the inversion. However, just below the inversion the rate of energy dissipation increases sharply.

In another study Ellsaesser (1969) presented ε in terms of the vector standard deviation and mean scalar winds. Using long term (1959-1962) climatological data he obtained the following values for the North American continent.

Height (mb)	Height (km)	ε (cm^2/sec^3)
50	20.9	1.623
100	16.6	2.455
200	12.4	6.600
300	9.7	8.591
500	5.9	4.832
700	3.2	2.190
850	1.5	2.284

Using Ellsaesser's results and the result of others, a suggested composite profile of ε with height is presented in Figure 2. Note that a peak occurs at a level just below the tropopause. Although different

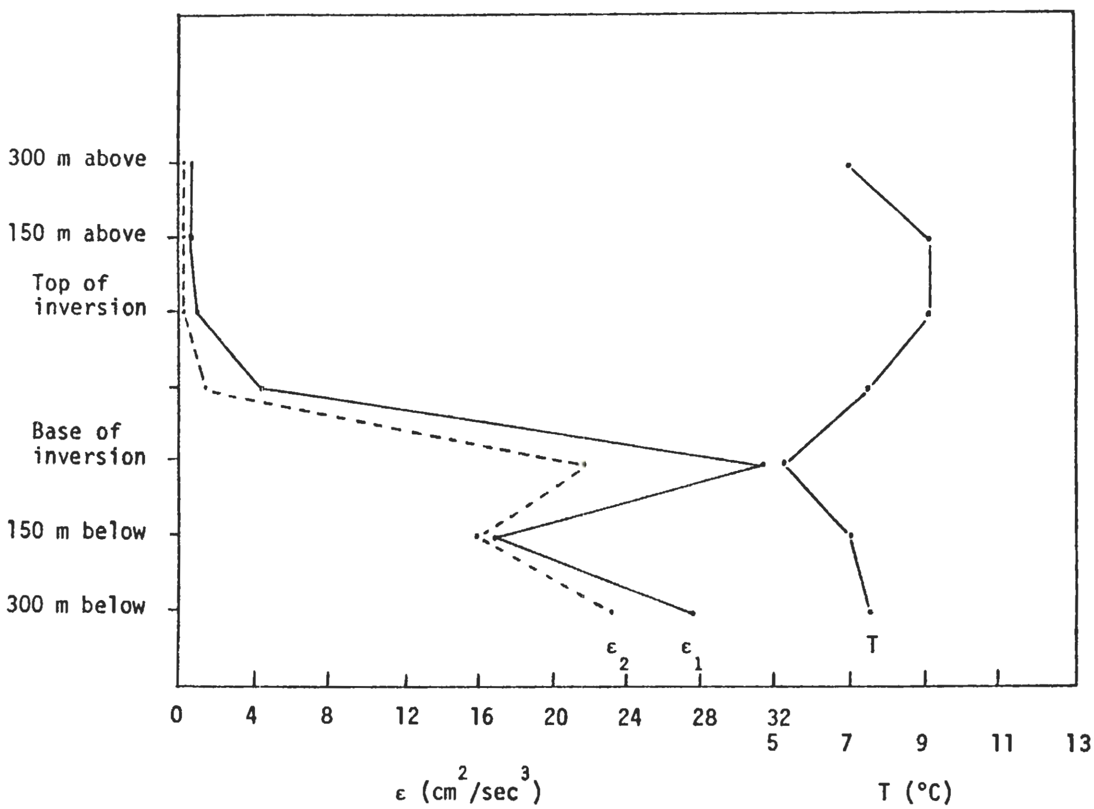

Figure 1. Variation of the Rate of Dissipation Through an Inversion (Readings and Rayment, 1969)

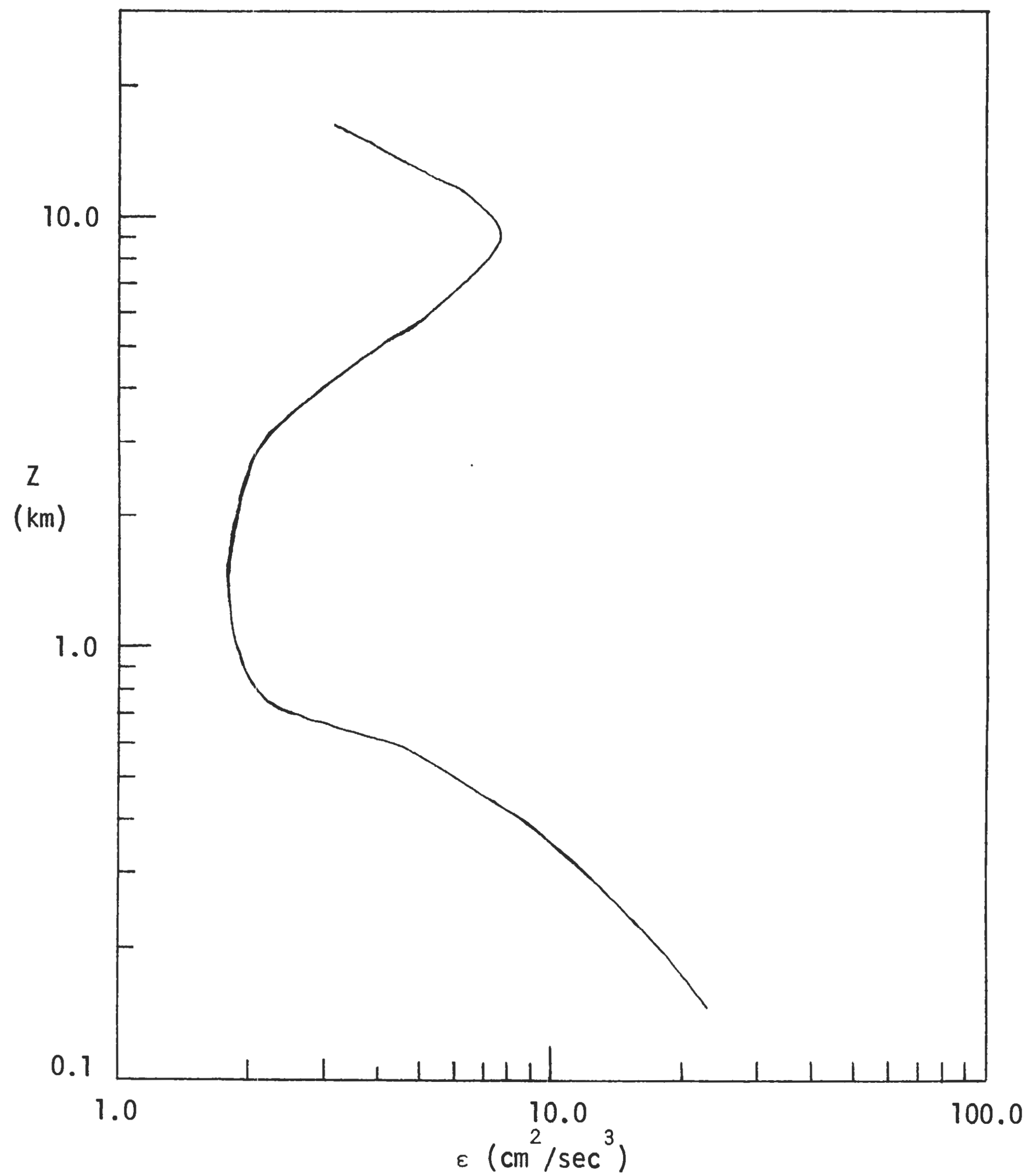

Figure 2. A Composite Profile of ε

scales are involved, this peak occurs in the same relative position as that observed by Readings and Rayment.

Estimates of the Eddy Diffusion Coefficient by Others

The eddy diffusion coefficient is not a quantity that can be measured directly. It can be estimated from observed data for parameters that can be measured directly. There have been several estimates of the diffusion coefficient using different techniques and data sets. These estimates suggest that the diffusion coefficient may have considerable variance.

One of the early estimates of the diffusion coefficient was made by G. I. Taylor (1915). Taylor examined the virtual-temperature vs time distribution over the Grand Banks off Newfoundland. From this he was able to make estimates of the diffusion coefficient

Table 3. Taylor's Estimates of K (Taylor, 1915)

Height (m)	K (m^2/sec)	Beaufort Wind Scale
610	0.15	2 - 3
270	0.34	3.3
200	0.25	3.0
170	0.13	2.2
140	0.057	2.0

H. J. Steward (1945) using data collected by G. M. B. Dobson (1914) examined the relationship between the Ekman spiral and viscous stresses. He obtained the following values for the diffusion coefficient:

Height (m)	K (m^2/sec)
900	4.7
800	3.7
600	2.0

The order of magnitude difference between Taylor and Stewart can be explained, in part, by the more stable maritime air encountered by Taylor. This comparison emphasizes the variability K can have.

Richard J. Reed and Kenneth E. German (1965) examined ozone and sensible heat fluxes in the stratosphere. Modifying the equation of motion through a form of Fickian diffusion, they obtained the following values for 30° N. latitude.

Table 4. Calculated Diffusion Coefficients (m^2/sec) Based on Heat Flux Data (Reed and German, 1965)

Height (km)	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec
24	0.20	0.06	0.14	0.15
20	0.51	0.14	0.16	0.37
16	3.10	1.51	0.93	2.37

Their computed values compared reasonably well with the Project Hardtack nuclear test series of 1958. In those tests tungsten 185 was injected into the lower stratosphere and was used as a tracer.

Following up on the efforts of Reed and German, Gudiksen et al. (1968) also examined the distribution of the tungsten 185 tracer. Gudiksen et al. obtained values of the diffusion coefficient from the variance of the meridional wind component. However, these values of K were found to be too large to explain the long term distribution of the tungsten 185. By applying scale factors to reduce the coefficients, more reasonable results were obtained. The reduced coefficients used in their modeling for 30° N lat. are:

Table 5. Calculated Diffusion Coefficients (m^2/sec) Based on Meridional Wind Variance (Gudiksen, et al., 1968).

Height (km)	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec
27	0.11	0.07	0.10	0.07
21	0.51	0.14	0.14	0.26
15	3.96	1.63	1.23	2.36

Radon decay products have also been used as a tracer. I. L. Karol (1966) of the Soviet hydrological service obtained aircraft measurements of lead-210(RaD) and polonium-210(RaF). From these measurements he was able to obtain estimates of the diffusion coefficient. The following values apply for 35-45° N. Lat.

Date	Layer (km)	K (m^2/sec)
Sep 1961	15 - 21	2.00
Oct 1961	15 - 20	0.24
Dec 1962	15 - 20	0.23
Jun 1962	15 - 21	3.40

The efforts of Reed and German and that of Karol are primarily for the lower stratosphere. Considering both the troposphere and the stratosphere, Davidson et al. (1966) examined the distribution of tungsten 185 from the 1958 U. S. nuclear tests as well as the strontium 90 distribution from the 1962 Soviet nuclear tests. From their modeling they found the following values of diffusion coefficient to give good agreement with observed data.

Layer	K (m^2/sec)
Stratosphere	0.1
Troposphere	4.0

Using Observed concentrations of methane taken by Ehhalt (1972) and Ehhalt et al. (1972), Wofsy and McElroy (1973) were able to model atmospheric mixing in the vertical. One of the better estimates of the eddy coefficient profile obtained by Wofsy and McElroy for the observed CH_4 data is

Height (km)	K (m^2/sec)
40	11.0
30	4.5
20	0.68
16	0.20
10	30.0
0	30.0

Radon (Rn^{220} and Rn^{222}) is exhaled from the earth's surface. The resulting vertical spreading of radon is caused by convection and turbulent diffusion. Modeling the distribution of radon and its decay products, Jacobi and Andre' (1963) obtained several profiles for the diffusion coefficient. These profiles differed according to meteorological conditions. The profile considered by them as representative of the normal turbulent conditions and the estimated range of values the coefficient could take are presented in Table 6.

However, some caution should be used before accepting the profile Jacobi and Andre' present as typical. This profile is based on fitting only two observed profiles of radon and assuming a source strength for radon. Thus the typical profile is "typical" in the sense that it may occur. Two soundings are insufficient in number to establish a mean or modal profile of K.

Table 6. Estimates of the Turbulent Diffusion Coefficient (Jacobi and Andre', 1963)

Height (km)	Typical (m^2/sec)	Range (m^2/sec)
30	0.2	0.2 - 0.2
20	0.6	0.4 - 1.0
10	15.0	1.7 - 90
8	18.0	2.0 - 100
6	20.0	2.0 - 100
4	20.0	2.0 - 100
2	19.0	1.5 - 100
1	17.0	1.0 - 100
0.1	10.0	0.1 - 70

All of these data indicate that the diffusion coefficient increases with altitude to the vicinity of or just below the tropopause. Here the coefficient decreases with height and maintains a low value through the lower stratosphere. At higher altitudes there is evidence not presented here that shows the diffusion coefficient to again increase with height.

CHAPTER IV

THE DATA USED IN THIS STUDY

The Data

The data used in this study are radiosonde wind and temperature measurements. The observations were taken by the National Weather Service, U. S. Department of Commerce. The radiosondes were launched from Athens, Georgia. After the completion of a sounding, the data were transferred to the National Climatic Center, Ashville, North Carolina for archival purposes.

A magnetic tape containing three months soundings was obtained from the National Climitic Center. The tape was created using the COBOL language with an IBM character set on a RCA machine maintained by Univac. The period of record on the tape was August, September, and October, 1974. The tape contained both the 00z and 12z soundings. Both standard and significant levels for each sounding are contained on the tape. However, only the standard levels were used in this study.

As far as this study is concerned, there is nothing unique in the choice of either the location or period of record. The choice was influenced only by the availability of data, and the desire to obtain data in the vicinity of Atlanta for possible use in other unrelated studies. With the exception of these restrictions, both the location and period of record may be considered as taken at random.

Determination of Longitudinal and Lateral Flow

In a laboratory situation the determination of longitudinal and lateral directions of flow is relatively simple. The longitudinal direction in this case may be taken as the direction parallel to the axis of the pipe, channel, or wind tunnel used. However, in the atmosphere there are not usually such convenient physical utensils available. In the atmosphere the wind can and does blow from almost every direction. Therefore the problem becomes one of the determining by some method an average or prevailing direction. This prevailing direction may then be assigned as the direction parallel to the longitudinal flow.

The method used in this study was to first construct wind roses for each of the 23 levels examined. From these wind roses an estimate of the most frequently occurring direction was made. In an effort to provide a check on the estimate, the skewness of the lateral wind distribution was examined. The objective was to find a direction that made the skewness as close to zero as possible. The logic is that for a normal distribution the skewness about the mean is zero. Table 7 shows the estimates used in this study. It should be pointed out the skewness values obtained are not precisely zero. However, the differences are felt to be within acceptable limits for experimental error.

The prevailing directions listed in Table 7 follow the meteorological convention of presenting wind direction in degrees east of north from which the wind is blowing. An interesting observation concerns the surface winds. Annually the prevailing surface wind for Athens, Georgia is from the northwest. However, the prevailing directions from long term data for the months covered in this study are:

Table 7. General Meteorological Data

Level (mb)	Pressure Height (m)	Prevailing Direction (deg)	Lateral Wind Skewness	Average Temperature (°C)	Longitudinal Wind Speed (m/sec)
20	26778	90	-0.11	-49.7	6.6
30	24132	90	0.27	-53.2	5.9
50	20884	90	0.03	-58.2	2.4
70	18801	280	-0.66	-63.2	1.3
100	16642	280	-0.22	-68.4	7.1
150	14216	270	0.00	-65.2	14.6
200	12405	270	-0.14	-56.5	16.8
250	10926	270	-0.14	-46.3	15.3
300	9667	270	-0.03	-36.7	13.4
350	8564	260	0.28	-28.3	11.6
400	7580	270	0.22	-21.2	10.4
450	6690	270	0.22	-15.0	9.1
500	5874	280	-0.03	-9.7	8.0
550	5123	280	0.11	-5.2	6.7
600	4427	280	-0.06	-1.3	5.9
650	3778	280	-0.11	2.1	5.0
700	3168	270	0.02	5.3	4.2
750	2593	270	-0.12	8.5	3.4
800	2050	236	0.18	11.0	2.2
850	1534	292	-0.53	13.5	1.2
900	1042	270	-0.14	16.7	0.3
950	571	68	-0.59	19.3	0.7
Surface	--	45	-0.36	19.6	0.5

Pressure height is taken from the U. S. Standard Atmosphere Supplements, 1966, for the month of July at 30° N Lat.

August	SW
September	NE
October	NE

This compares well with the direction of 45° obtained from the radiosonde data.

A Test for Isotropy

In the theory of isotropic turublence the axes are invariant with rotation. From Kolmogrov it might be assumed the probability distributions are independent with respect to the axes. It is suggested this could be the basis of a test for isotropy.

Using the Central Limit Theorem it is possible to assume the fluctation components along each of the axes have a normal distribution. That is,

$$V_i' \sim N(\mu_i, \sigma_i^2) \quad i = 1, 2, 3$$

If the σ_i^2 are not equal then it is postulated the turbulence is anisotropic.

The radiosonde data used in this study does not provide any information about the fluctuations in the vertical. Thus only two dimensional isotropy will now be considered. The logic is that while the three dimensional case can not be proven here, the three dimensional case can not exist if the two dimensional case fails.

The hypothesis to be tested is:

$$H_0: \quad \sigma_n^2 = \sigma_1^2$$
$$H: \quad \sigma_n^2 \neq \sigma_1^2$$

This can be done by a two-sided F test on the sample variances S_n^2 and S_1^2. Letting $\nu = N_o - 1$, where N_o is the number of observations, the test statistic is

$$F_0 = \frac{S_1^2}{S_n^2} \sim F_{\nu,\, \nu}$$

H_0 would not be rejected if

$$\frac{1}{F_{\alpha/2,\, \nu,\, \nu}} \leq F_0 \leq F_{\alpha/2,\, \nu,\, \nu}$$

where α is the probability of a type I error. That is α is the probability of rejecting H_0 when H_0 is really true.

It is also possible to determine the power of such a test. If $\Lambda = \sigma_1^2/\sigma_n^2$, then the probability of a type II error is

$$\beta = P\left\{\frac{F_{1-\alpha/2,\, \nu,\, \nu}}{\Lambda} \leq F_0 \leq \frac{F_{\alpha/2,\, \nu,\, \nu}}{\Lambda}\right\}$$

β is the probability of failing to reject the null hypothesis when H_0 is actually false. The power of the test or the probability of correctly rejecting H_0 is then

$$1 - \beta$$

While it is possible to actually compute this value an easier way is to utilize Operating Characteristic (OC) curves. For this study the curves appearing in Hines and Montgomery (1972) were used. There are around 180 observations per level in this study. For a Λ of about 1.4

the OC curves indicate a probability around 90% of correctly rejecting H_0 at the 5% level of significance and a probability of about 85% at the 1% level. As a result of the sample sizes used, this would seem to be a good test.

F_0 has been computed for each of the standard levels looked at this study. These values are shown in Table 8. The values of the F distribution may be found in many statistics books. The tables appearing in Hines and Montgomery (1972) were used here.

An examination of Table 8 reveals the fact that the longitudinal and lateral variances differ significantly from each other near the surface and in the stratosphere. Thus the turbulence may be considered anisotropic at these levels.

The fact that turbulence is anisotropic near the surface is not unexpected. Near the surface there is not sufficient room for the eddies to develop in equal size in all directions. The earth's surface has the effect of anisotropically damping the eddies.

The anisotropic turbulence in the stratosphere can be explained by the much greater stability existing in that region. This stability has the effect of damping vertical fluctuations.

Table 8. Test for Isotropic Turbulence

Level (mb)	S_1 (m/sec)	S_n (m/sec)	Number of Observations	Test Statistic $F_0 = S_1^2/S_n^2$
20	8.92	2.21	134	16.29 *
30	7.03	2.30	158	9.34 *
50	6.13	1.99	170	3.08 *
70	5.24	2.85	174	3.38 *
100	7.50	4.22	178	3.16 *
150	10.20	8.47	180	1.45 *
200	17.10	12.20	181	0.98
250	11.70	11.80	182	0.98
300	10.60	10.00	182	1.12
350	9.37	8.66	182	1.17
400	8.06	7.81	182	1.06
450	7.14	6.81	182	1.10
500	6.40	6.31	182	1.03
550	5.95	5.85	182	1.03
600	5.86	5.62	182	1.09
650	5.06	5.34	180	0.90
700	4.57	4.91	180	0.87
750	4.27	4.71	180	0.82
800	4.45	4.10	180	1.18
850	4.10	3.86	179	1.13
900	4.51	3.17	180	2.02 *
950	4.84	2.78	180	3.03 *
Surface	1.60	1.32	184	1.47 *

* F_0 is significant at both the 5% and 1% levels, therefore the hypothesis that three dimensional isotropy exists at these levels is rejected.

CHAPTER V

DETERMINATION OF ε AND K_m

The rate of turbulent energy dissipation ε was calculated in this study by computing the longitudinal structure function

$$D_{11}(\tau) = \overline{[u_1(t + \tau) - u_1(t)]^2}$$

$D_{11}(\tau)$ was then plotted vs τ on a log-log scale. The slope m was then estimated for that region where the slope was approximately 2/3. As was shown earlier.

$$D_{11}(\tau) \propto \tau^m$$

The constant of proportionality was taken to be 2.14ε where 2.14 is an assumed value for the Kolmogrov constant. Thus ε was estimated by

$$\varepsilon = \frac{D_{11}(\tau)}{2.14\ \tau^m}$$

where τ was taken from values within the 2/3 range. The estimated values of m and ε obtained are shown in Table 9.

The eddy diffusivity coefficient K_m was calculated using the Lilly formula in the form

$$K_m = 1/4\ \varepsilon/\overline{N}^2$$

where $\overline{N}$ is the average Brunt-Vaisala frequency. Individual Brunt-Vaisala

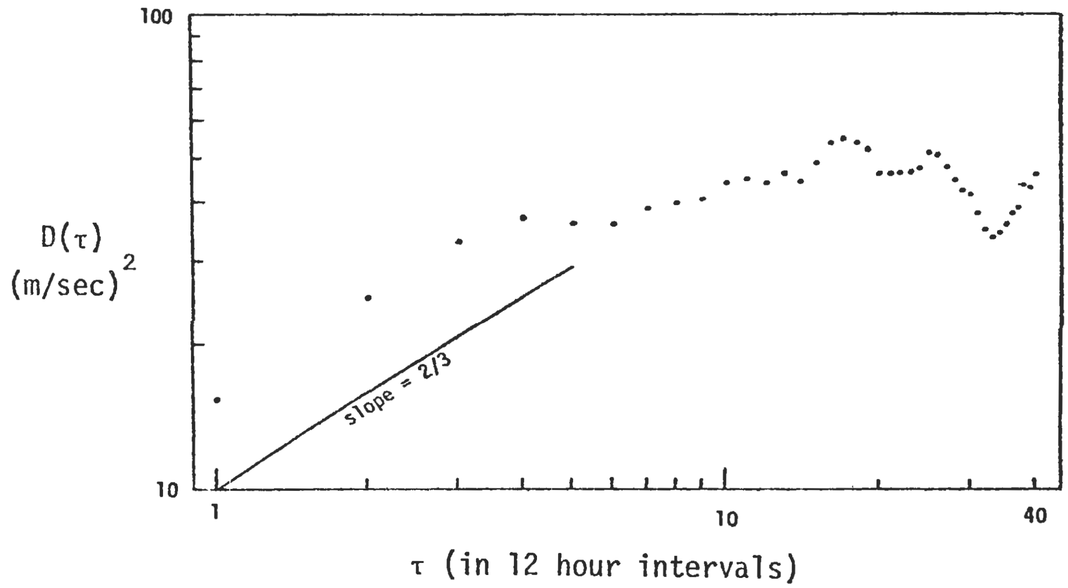

Figure 3. Longitundinal Structure Function - 700 mb Winds

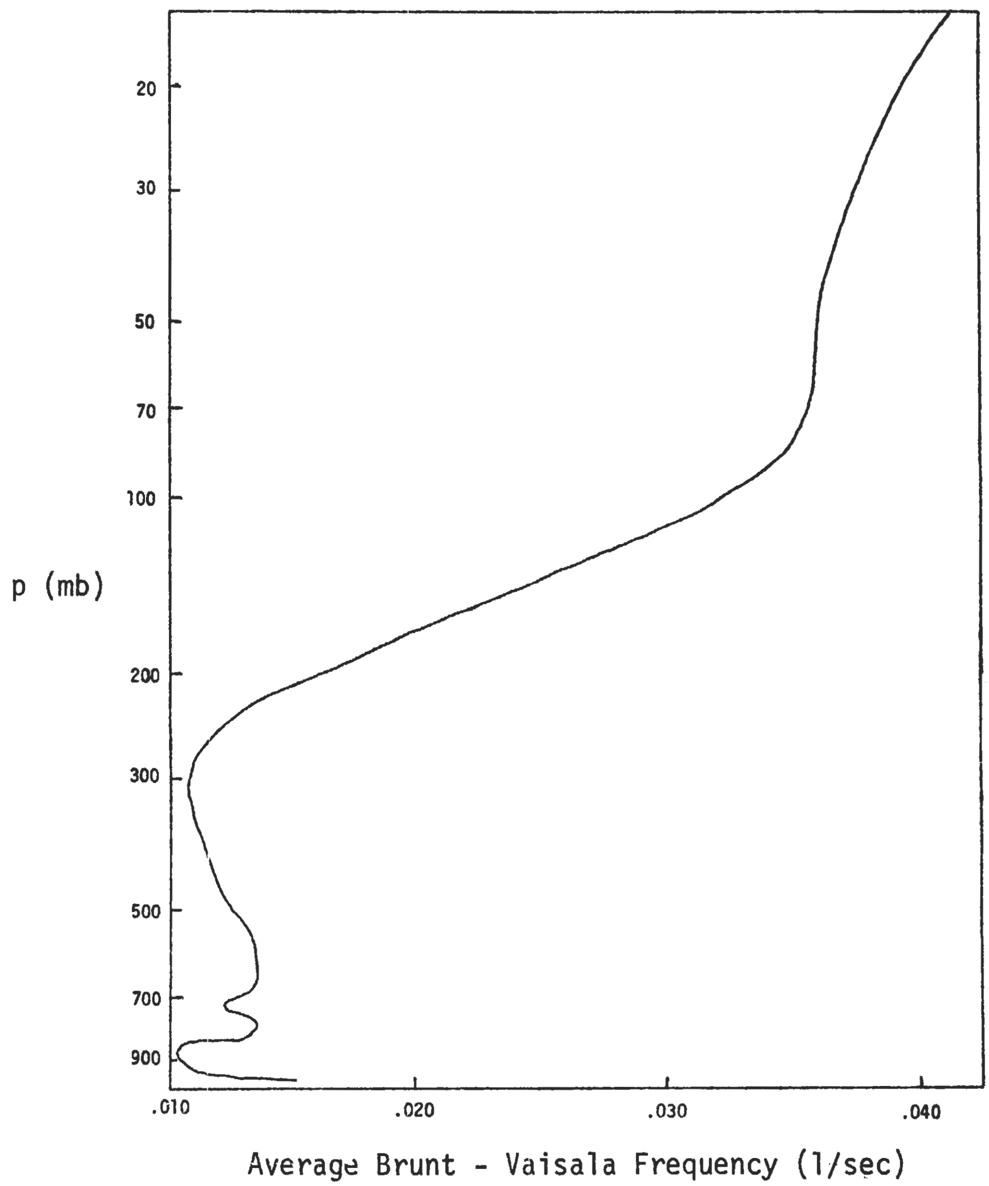

Figure 4. Average Brunt-Vaisala Frequency Computed from Athens, Georgia, 00z and 12z Radiosonde Soundings, Aug.-Oct., 1974

frequencies for each sounding were calculated by

$$N = \left(\frac{g}{T} \frac{\Delta\theta}{\Delta z} \right)^{1/2}$$

The average of these individual frequencies was then obtained. $\overline{N}$ as calculated was for a layer of thickness Δz and not for a specified level. To overcome this difficulty, the computed values of $\overline{N}$ were plotted at the midpoint of each Δz and a smooth curve was then constructed connecting the plotted midpoints. Estimates of $\overline{N}$ were then taken from this curve (Figure 4) for each of the levels of interest. The values obtained in this manner are shown in Table 9.

Using the estimated values of ε and $\overline{N}$, values of K_m were in turn estimated. These values of K_m are shown in Table 9.

The essential results of this study are illustrated in Figure 5 and Figure 6. These show the profiles of ε and K_m in the vertical. Due to possible sampling problems, values in the surface boundary layer are not drawn.

The values for ε in Figure 5 above the surface boundary layer show a somewhat similar shape to the composite profile in Figure 2. The values in Figure 5 are somewhat larger. Both figures show a maximum at or below the tropopause. It is felt that if data were taken with a time interval much shorter than 12 hours, then an increase in the boundary layer might be observed.

The profile of K_m suggests a near constant K_m with height above the boundary layer and below the turbulent layer under the tropopause. The values obtained for the troposphere suggest some validity to the mo-

Table 9. Estimated Values for ε, Brunt-Vaisala Frequency, and K_m

Level (mb)	Slope m	$D_{11}(\tau)/\tau^m$	ε (cm^2/sec^3)	Brunt-Vaisala Frequency $\bar{N}$, (1/sec)	K_m (m^2/sec)
20	.77	4.44	2.1	0.0394	0.033
30	.55	5.33	2.5	0.0375	0.044
50	.52	6.29	2.9	0.0360	0.057
70	.57	7.09	3.3	0.0356	0.065
100	.71	12.52	5.8	0.0323	0.14
150	.68	98.29	46.0	0.0230	2.2
200	.75	60.49	28.0	0.0162	2.7
250	.70	60.40	28.0	0.0121	4.8
300	.47	74.29	35.0	0.0108	7.4
350	.99	38.96	18.0	0.0110	3.8
400	.97	26.58	12.0	0.0115	2.3
450	1.07	21.09	9.8	0.0120	1.7
500	.78	20.42	9.5	0.0127	1.5
550	.64	22.44	10.0	0.0133	1.5
600	.68	21.80	10.0	0.0134	1.4
650	.46	22.90	11.0	0.0134	1.5
700	.67	15.60	7.3	0.0138	1.1
750	.88	13.00	6.1	0.0130	0.90
800	.72	13.20	6.2	0.0135	0.85
850	.54	17.00	7.9	0.0105	1.8
900	.76	14.60	6.8	0.0105	1.5
950	.46	19.57	9.1	0.0121	1.6
Surface	.54	1.89	0.88	0.0155	0.09

Where $\varepsilon = \frac{1}{2.14} D_{11}(\tau)/\tau^m$ and $K_m = \frac{\varepsilon}{4\bar{N}^2}$

del presented by Davidson et al., (1966) where the diffusivity was a constant in the troposphere. The values obtained in this study are somewhat smaller than that modeled by Davidson et al. Since there is some uncertainty in the assumptions of the Lilly formula, it is felt that the values represent an approximate three month mean for the chosen location to within a factor of 2.

An important assumption that was made was that the buoyancy effects were minimal. If, as Lilly assumes, the buoyancy effect is 1/3 the dissipation and not 1/10 as assumed have, then all of the K_m values presented here are low by a constant factor.

A thorough examination of the effects of finite sampling and averaging times is beyond the scope of this work. For this study the effect of finite sampling and averaging times is that of a filter. That is, certain fluctuations are eliminated by the data collection process and are not available for analysis. As these fluctuations are eliminated, the sample variance decreases. Thus to keep the sample variance close to the whole variance obtained with infinitesimal averaging time, sample sets should be as large as possible, since the one minute averages of the radiosonde are not subject to easy change.

An additional problem is the fact that the radiosonde observations are 12 hours apart and the averaging time is on the order of one minute. Thus only 0.139% of the turbulence is actually observed. Events occurring between observations go unrecorded and thus unprocessed. Thus many short lived fluctuations are not included in this study.

The values for ε and the resulting K_m near the surface appear to be on the low side. It is felt that the cause for the relatively low values lies with the data collection process rather than a failure of the theory. The time constant of 12 hours between radiosonde soundings filters out the short lived turbulence existing near the surface. Another factor is that the soundings are made at 7:00 A. M. and 7:00 P. M. EST. daily. At these times turbulence is not fully developed near the surface while the levels above the surface are more affected by synoptic influences. Consequently the energy dissipation is greatly reduced near the surface at these times of the day.

The energy dissipation maximum and hence rapid diffusion rate just below the tropopause is analogous to the observation by Readings and Rayment (1969) of an energy dissipation maximum just below an inversion. Readings and Rayment suggested the cause of this ε maximum to be due to the decay of thermal energy. In this situation a smoke plume could rise relatively intact to a turbulence layer below an inversion. Upon reaching the turbulent layer the plume would spread horizontally rather rapidly. This can be seen on a larger scale in the atmosphere when observing a thunderstorm. The convective cell (plume) remains relatively intact with observable boundaries. Near the tropopause the thunderstorm spreads out in a vertically thin, horizontally broad anvil downwind from the thunderstorm cell (plume). The difference here between the smoke plume and the thunderstorm is one of scale as far as the atmospheric processes are concerned.

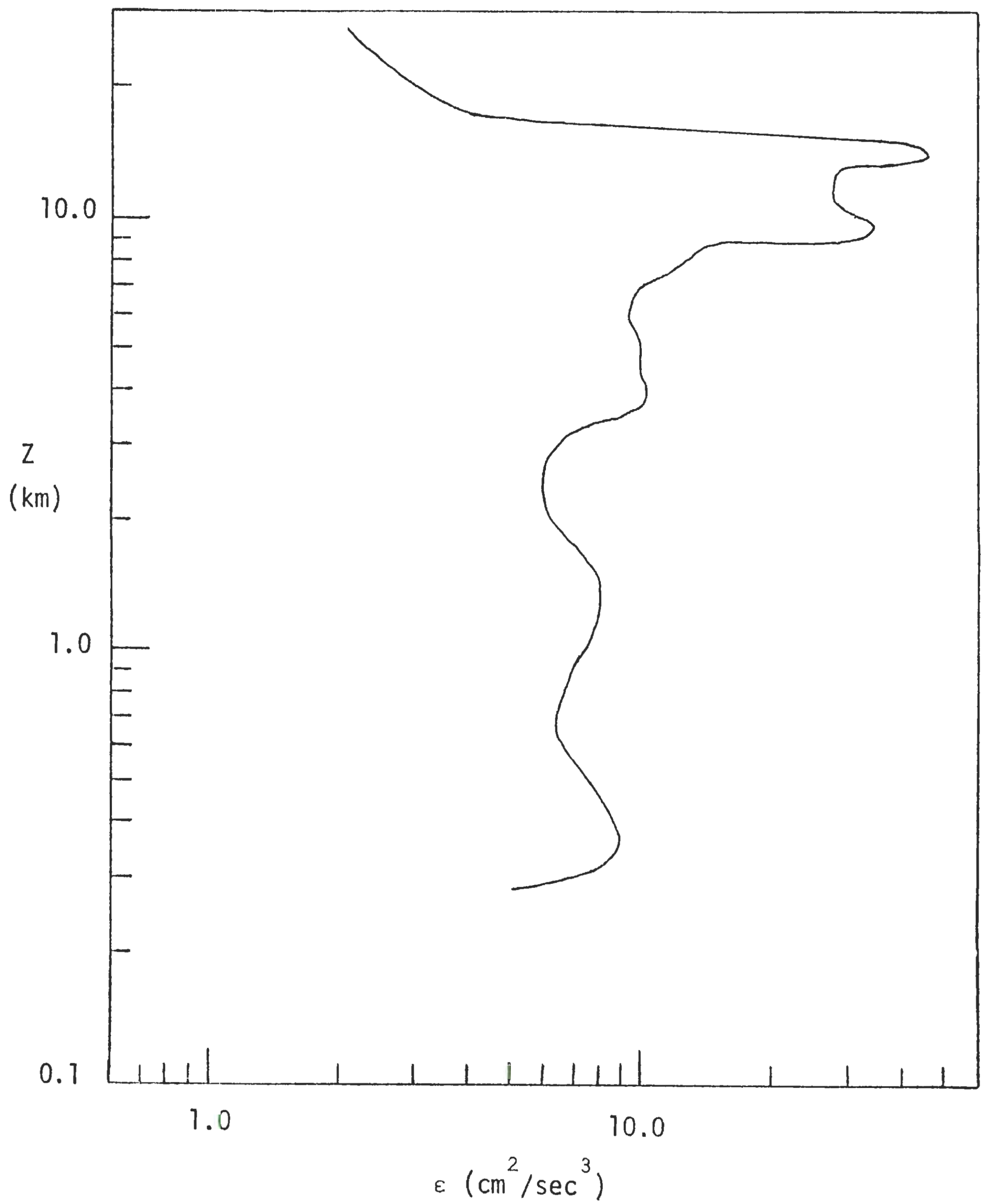

Figure 5. A Profile of ε From Radiosonde Data

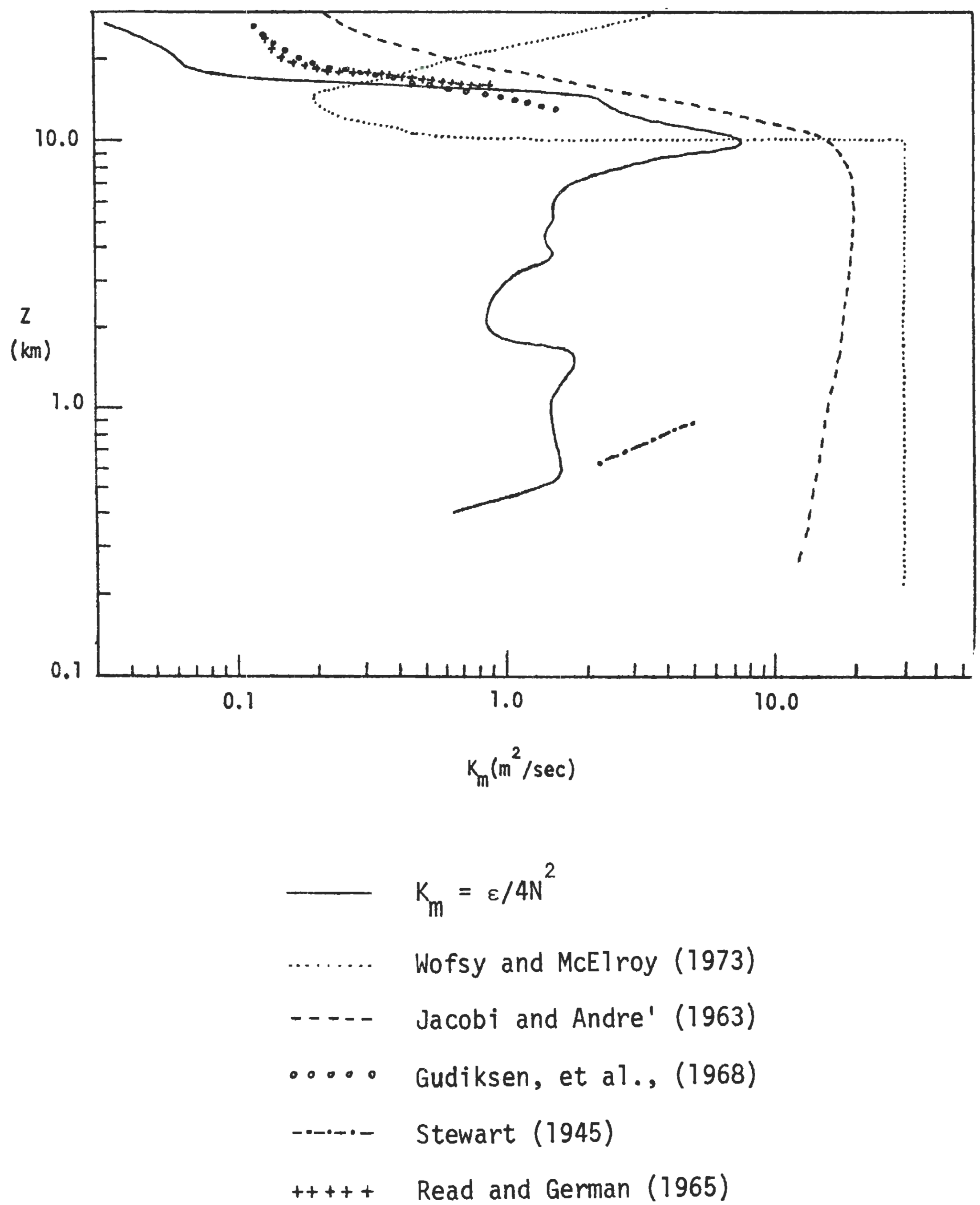

Figure 6. A Profile of K_m From Radiosonde Data

CHAPTER VI

CONCLUSIONS

It is felt that the Kolmogrov Similarity Theory of energy dissipation and the Lilly formula provide a means for examining relative features of the atmospheric diffusion processes. The technique as presented in this paper allows the diffusion coefficient to be estimated directly from routinely collected meteorological data.

In any diffusion study the question of scale needs to be considered. In this study there existed a relatively large time interval between observations. As a result only gross features existing over a three month period were examined. If data with a shorter time interval between observations were available, then features of a smaller time scale could be observed. Likewise, if data were available at more levels, then more details of the vertical structure could be observed. It is the data that impose restrictions on the scales examined and not the method.

The method presented in this paper does provide some basis for considering diffusion as an energy process. Thus a knowledge of ε may be used to estimate the rate at which diffusion is taking place. By using an energy approach, restrictions on the physical nature of the material being diffused are kept to a minimum.

The method presented in this paper suggests that the diffusion coefficient in the atmosphere is not a constant and vertical profiles of K_m indicate details of considerable interest. It is felt that the K_m

peak just under an inversion is not a chance occurrence seen only in the examples presented in this paper. It is felt that the observed peak plays a role in the atmospheric processes. Further studies using this method should provide some insight to its role.

The structure function allows for a much greater utilization of existing data compared to other methods. By taking advantage of this greater utilization greater details may be observed in mathematical form. The use of the structure function permits the calculation of a rate of energy dissipation. The rate of energy dissipation can be used to estimate the rate of diffusion taking place. Thus the basis for the method presented here is that diffusion is an energy process. Therefore if knowledge of the energy processes is known, then the diffusion can be estimated.

APPENDIX

The following is a listing of the program used to calculate the values of the structure function. The program was written in the FORTRAN language for use on a Control Data Corporation CDC 7000 series computer. Generous use was made of subroutines throughout the program. The computer results were verified by checking against hand calculations for a sample set of data prior to running over the entire Athens data. Since missing data values are a real world problem, the program was designed with recognition of missing data in mind.

The control cards for use of the program are (omitting user number and password cards):

```
REQUEST(TAPE1,MT,PO=R,VSN-X0088)
FTN.
LGO.
```

```
      PROGRAM MAINBOD (INPUT,OUTPUT,TAPE1,TAPE6=OUTPUT)


      DIMENSION DIR(30)
      COMMON /BLOCK1/ IBUF(200)
      COMMON /BLOCK2/ P(79), Z(79), T(79), RH(79), WD(79),
     +       WS(79)
      COMMON /BLOCK3/ PS(30), ZS(30), TS(30), RHS(30),
     +       WDS(30), WSS(30)
      COMMON /BLOCK6/ AMONTH(13)
      COMMON /BLOCK9/ PMISS,ZMISS,TMISS,RHMISS,WDMISS,WSMISS
      COMMON /BLOCK10/ D(30,40)
      COMMON /BLOCK11/ U(30,184)
      DATA AMONTH/9H JANUARY ,9HFEBRUARY ,9H  MARCH  ,
     +       9H  APRIL  ,
     +    9H   MAY   ,9H  JUNE   ,9H  JULY   ,9H AUGUST  ,
     +       9HSEPTEMBER,
     +    9H OCTOBER ,9HNOVEMBER ,9HDECEMBER ,9H ANNUAL  /
      DATA ISTAT,IYR,IMO,IDAY,IHR,NLVL/6*0/
      DATA PMISS, ZMISS, TMISS, RHMISS/0.0,-99999.,-99.9,
     +       -999./
      DATA WDMISS, WSMISS/999.,999./
      DATA DIR/2*45.,68.,270.,292.,236.,2*270.,4*280.,
     +       2*270.,260.,
     +     4*270.,2*280.,3*90.,270.,249.,4*999./
      WRITE(6,6000)
      IM1= 8
      IM2= 10
      IHOUR= 25
      IYR= 74
      I = 0
      REWIND 1
   10 CONTINUE
      BUFFER IN (1,1) (IBUF(1), IBUF(200))
      IF(UNIT(1)) 30, 600, 20
   20 WRITE(6,6010)
      GO TO 900
   30 CONTINUE
      CALL PLUCK(IYR,IMO,IDAY,IHR,NLVL)
      CALL UNPAK(NLVL)
      CALL STANLVL
      IF(IHR.EQ.0.OR.IHR.EQ.12) 40, 10
   40 CONTINUE
      I = I + 1
      DO 50 L=1,30
      DEG = 0.0174532925*(WDS(L) - DIR(L))
      U(L,I) = WSS(L)*COS(DEG)
```

```
      PROGRAM MAINBOD (INPUT,OUTPUT,TAPE1,TAPE6=OUTPUT)

      IF(WSS(L).EQ.WSMISS.OR.WDS(L).EQ.WDMISS) U(L,
     +       I) = WSMISS
   50 CONTINUE
      GO TO 10
  600 CONTINUE
      DO 610 L=1,30
      CALL STRUCT(L,WSMISS)
      CALL RITE7(IM1,IM2,IYR,L,40)
  610 CONTINUE
  900 CONTINUE
      REWIND 1
      STOP
 6000 FORMAT(1H1)
 6010 FORMAT(1H0,*PARITY ERROR*)
      END
```

```
      SUBROUTINE PLUCK (IYR,IMO,IDAY,IHR,NLVL)
C
C        THIS SUBROUTINE REQUIRES THE USE OF
C             INTEGER FUNCTION NGET
C             SUBROUTINE IGET
C        TDF-56 FORMAT IS ASSUMED
C
      COMMON /BLOCK1/IBUF(200)
      IS=5
      IE= IS + 4
      ISTAT= NGET(IS,IE)
      IS= IE + 1
      IE= IS + 1
      IYR= NGET(IS,IE)
      IS= IE + 1
      IE = IS + 1
      IMO= NGET(IS,IE)
      IS= IE + 1
      IE= IS + 1
      IDAY= NGET(IS,IE)
      IS= IE + 1
      IE= IS + 1
      IHR= NGET(IS,IE)
      IS= IE + 1
      IE= IS + 1
      NLVL= NGET(IS,IE)
      RETURN
      END
```

```
      SUBROUTINE UNPAK (NLVL)
C
C        THIS SUBROUTINE REQUIRES THE USE OF
C             INTEGER FUNCTION NGET
C             SUBROUTINE IGET
C        TDF-56 FORMAT IS ASSUMED
C
      COMMON /BLOCK1/IBUF(200)
      COMMON /BLOCK2/ P(79), Z(79), T(79), RH(79), WD(79),
     +        WS(79)
      COMMON /BLOCK9/ PMISS,ZMISS,TMISS,RHMISS,WDMISS,WSMISS
      DO 10 J=1,NLVL
      IS=J*25 + 1
      IE= IS + 4
      P(J)= FLOAT(NGET(IS,IE))/10.0
      IS= IE + 1
      IE= IS + 4
      Z(J)= FLOAT(NGET(IS,IE))
      IS= IE + 1
      IE= IS + 2
      T(J)= FLOAT(NGET(IS,IE))/10.0
      IS= IE + 1
      IE= IS + 2
      RH(J)= FLOAT(NGET(IS,IE))
      IS= IE + 1
      IE= IS + 2
      WD(J)= FLOAT(NGET(IS,IE))
      IS= IE + 1
      IE= IS + 2
      WS(J)= FLOAT(NGET(IS,IE))
   10 CONTINUE
      IF(NLVL.LT.79) GO TO 20
      RETURN
   20 LVL= NLVL + 1
      DO 30 J=LVL,79
      P(J)= PMISS
      Z(J)= ZMISS
      T(J)= TMISS
      RH(J)= RHMISS
      WD(J)= WDMISS
      WS(J)= WSMISS
   30 CONTINUE
      RETURN
      END
```

```
      INTEGER FUNCTION NGET (IS,IE)


C
C        THIS INTEGER FUNCTION REQUIRES THE USE OF
C             SUBROUTINE IGET
C
      COMMON /BLOCK1/IBUF(200)
      NGET= 0
      DO 30 J=IS,IE
      CALL IGET(J, IGOT, ISIGN)
      NGET= IABS(NGET) + IGOT
      NGET= ISIGN*NGET
      IF(J.LT.IE)  NGET= NGET*10
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE IGET(N, IGOT, ISIGN)

      COMMON /BLOCK1/IBUF(200)
      ISIGN= 1
      IWORD= (N-1)/10 + 1
      IC= MOD(N-1,10)
      IC= IC*6
      IX= SHIFT(IBUF(IWORD), IC)
      IX= AND(IX, MASK(6))
      IX= SHIFT(IX,6)
      IF(IX.GE.27.AND.IX.LE.36)  10, 20
   10 IGOT= IX - 27
      GO TO 110
   20 IF(IX.LE. 9)  30, 40
   30 IGOT= IX
      GO TO 110
   40 IF(IX.LT.27)  50, 60
   50 IGOT= IX - 9
      ISIGN= -1
      GO TO 110
   60 IF(IX.EQ.58)  70, 80
   70 IGOT= 0
      GO TO 110
   80 IF(IX.EQ.54)  90, 100
   90 IGOT= 0
      ISIGN= -1
      GO TO 110
  100 WRITE(6,6010)
      IGOT= IX
  110 CONTINUE
      RETURN
 6010 FORMAT(1H ,*PROBLEM IN SUBROUTINE IGET*)
      END
```

```
      SUBROUTINE STANLVL

      COMMON /BLOCK2/ P(79), Z(79), T(79), RH(79), WD(79),
     +        WS(79)
      COMMON /BLOCK3/ PS(30), ZS(30), TS(30), RHS(30),
     +        WDS(30), WSS(30)
      COMMON /BLOCK9/ PMISS,ZMISS,TMISS,RHMISS,WDMISS,WSMISS
      DATA (PS(I), I= 2, 9)/1000.,950.,900.,850.,800.,750.,
     +       700.,650./
      DATA (PS(I), I=10,17)/600.,550.,500.,450.,400.,350.,
     +       300.,250./
      DATA (PS(I), I=18,28)/200.,150.,100.,70.,50.,30.,20.,
     +       10.,7.,5.,3./
      DATA (PS(I), I=29,30)/2.,1./
      PS(1)= P(1)
      ZS(1)= Z(1)
      TS(1)= T(1)
      RHS(1)= RH(1)
      WDS(1)= WD(1)
      WSS(1)= WS(1)
      JJ= 2
      DO 40 I= 2,30
      ZS(I)= ZMISS
      TS(I)= TMISS
      RHS(I)= RHMISS
      WDS(I)= WDMISS
      WSS(I)= WSMISS
      DO 20 J=JJ,79
      IF(P(J).EQ.PS(I)) 10,20
   10 ZS(I)= Z(J)
      TS(I)=T(J)
      RHS(I)= RH(J)
      WDS(I)= WD(J)
      WSS(I)= WS(J)
      GO TO 30
   20 CONTINUE
   30 JJ= J
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STRUCT(L,UMISS)
C
C        THIS SUBROUTINE COMPUTES THE STRUCTURE FUNCTION.
C
      COMMON /BLOCK10/ D(30,40)
      COMMON /BLOCK11/ U(30,184)
      DIMENSION SUM(30,40), N(30,40)
      DATA SUM,N/1200*0.0,1200*0/
      DO 10 LAG=1,40
      DO 10 I=1,184
      II = I + LAG
      IF(II.GT.184) GO TO 10
      IF(U(L,II).EQ.UMISS.OR.U(L,I).EQ.UMISS) GO TO 10
      SUM(L,LAG) = SUM(L,LAG) + (U(L,II) - U(L,I))**2
      N(L,LAG) = N(L,LAG) + 1
   10 CONTINUE
      DO 50 LAG =1,40
      IF(N(L,LAG).EQ.0) 30,40
   30 D(L,LAG) = -9.9
      GO TO 50
   40 D(L,LAG) = SUM(L,LAG)/FLOAT(N(L,LAG))
   50 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RITE7(IM1,IM2,IYR,L,LAG)
C
C        THIS SUBROUTINE WRITES OUT THE COMPUTED VALUES OF
C        THE STRUCTURE FUNCTION.
C
      COMMON /BLOCK3/ PS(30), ZS(30), TS(30), RHS(30),
     +          WDS(30), WSS(30)
      COMMON /BLOCK6/ AMONTH(13)
      COMMON /BLOCK10/ D(30,40)
      DIMENSION ARRAY(21,40), ADR(21)
      DATA AA, AB, AC/1HX,1H ,1H-/
      DATA ADR/21*5H    I/
      DATA A1,A2,A3,A4/5H 50 +,5H100 +,5H150 +,5H200 +/
      WRITE(6,6000)
      ADR(5) = A1
      ADR(10) = A2
      ADR(15) = A3
      ADR(20) = A4
      DO 10 K=1,LAG
      DO 10 I=1,21
      ARRAY(I,K) = AB
   10 CONTINUE
      DO 30 K=1,LAG
      IF(D(L,K).EQ.-9.9) GO TO 30
      IND = INT(D(L,K)/10.0) + 1
      IF(IND.GE.21) IND = 21
      DO 20 J=1,21
      IF(IND.EQ.J) ARRAY(J,K) = AA
   20 CONTINUE
   30 CONTINUE
      WRITE(6,6010) AMONTH(IM1), IYR, AMONTH(IM2), IYR
      IF(L.NE.1) GO TO 40
      WRITE(6,6020)
      GO TO 50
   40 WRITE(6,6030) PS(L)
   50 CONTINUE
      WRITE(6,6035)
      WRITE(6,6040)
      DO 60 K=1,20
      K1 = K
      K2 = K + 20
      WRITE(6,6050) K1,D(L,K1),K2,D(L,K2)
   60 CONTINUE
      WRITE(6,6060)
      DO 70 I=1,21
```

```
      SUBROUTINE RITE7(IM1,IM2,IYR,L,LAG)


      II = 22 - I
      WRITE(6,6070) ADR(II), (ARRAY(II,J), J=1,LAG)
   70 CONTINUE
      WRITE(6,6080) (M, M=5,40,5)
      RETURN
 6000 FORMAT(1H1,2(/))
 6010 FORMAT(1H ,60X,*ATHENS, GEORGIA*,/,
     +       1H0,60X,A9,*, 19*,I2,/,
     +       1H ,66X,*TO*,/,
     +       1H ,60X,A9,*, 19*,I2)
 6020 FORMAT(1H0,64X,*SURFACE*)
 6030 FORMAT(1H0,60X,F5.0,* MB LEVEL*)
 6035 FORMAT(1H ,62X,*LONGITUDINAL*)
 6040 FORMAT(1H ,80X,*2*,/,
     +       1H ,54X,*STRUCTURE FUNCTION (M/SEC) *,//,
     +       1H0,46X,2(* R *,5X,*D(R)*,8X))
 6050 FORMAT(1H ,46X,2(I2,3X,F9.3,6X))
 6060 FORMAT(1H0)
 6070 FORMAT(1H ,23X,A5,40(1X,A1))
 6080 FORMAT(1H ,27X,1H-,8(9(1H-),1H+),1H-,/,
     +       1H ,36X,8(I2,8X),/,
     +       1H ,57X,*LAG (12 HR INTERVALS)*)
      END
```

BIBLIOGRAPHY

Batchelor, G. K., 1950, Application of the Similarity Theory of Turbulence to Atmospheric Diffusion, Quarterly Journal of the Royal Meteorological Society, Vol. 76 (328), p. 133-146

________, 1951, The Application of the Similarity Theory of Turbulence to Atmospheric Diffusion, Quarterly Journal of the Royal Meteorological Society, Vol. 77 (332), p. 315-317.

________ and A. A. Townsend, 1948, Decay of Isotropic Turbulence in the Initial Period, Proceedings of the Royal Society of London, Series A, Vol. 193, p. 539-558

Bjerknes, V., Januar, 1904, Das Problem der Wettervorher sage, betrachtet vom Standpunkte der Mechanik und der Physik, Meteorologische Zeitschrift, Jahrgang 21, Band 39, p. 1-7 (English translation by Y. Mintz, Los Angeles, 1954).

Businger, J. A., 1968, On the Energy Supply of Clear Air Turbulence, Document D1-82-0740, Boeing Scientific Research Laboratory, Seattle, Washington.

Davidson, B., J. P. Friend, and H. Seitz, 1966, Numerical Models of Diffussion and Rainout of Stratospheric Radioactive Materials, Tellus, Vol. 18 (2), p. 301-315.

Dobson, G. M. B., 1914, Pilot Balloon Ascents at the Central Flying School, Upavon, During the year 1913, Quarterly Journal of the Royal Meteorological Society, Vol. 40 (170), p. 123-134.

Ehhalt, D. H., 1972, Carbon and the Biosphere, Brookhaven Symposium on Biology.

________, L. E. Heidt, and E. A. Martell, May 20, 1973, The Concentration of Atmospheric Methane Between 44 and 62 Kilometers Altitude, Journal of Geophysical Research, Vol. 78 (15), p. 2619-2624.

Ekman, V. Walfrid, 1911, On the Change from Steady to Turbulent Motion of Liquids, Arkiv For Matematik, Astronomi Och Fysik, Band 6, No. 12, p. 1-16.

Ellsaesser, Hugh W., June, 1969, A Climatology of Epsilon (Atmospheric Dissipation), Monthly Weather Review, Vol. 97 (6), p. 415-423.

Fick, Adolph, 1855, Uber Diffusion, Annalen der Physik und Chemie, Band 94, p. 59-86, also see On Liquid Diffusion, Philosophical Magazine, Vol. 4 (10), p. 30-39.

Gifford, Frank, Jr., October, 1957, Relative Atmospheric Diffusion of Smoke Puffs, Journal of Meteorology, Vol. 14 (5), p. 410-414.

Grant, H. L., R. W. Stewart, and A. Moilliet, February, 1962, Turbulence Spectra from a Tidal Channel, Journal of Fluid Mechanics, Vol. 12, part 2, p. 241-254.

Gudikesen, Paul H., A. W. Fairhall, and Richard J. Reed, July 15, 1968, Roles of Mean Meridional Circulation and Eddy Diffusion in the Transport of Trace Substances in the Lower Stratosphere, Journal of Geophysical Research, Vol. 73 (14), p. 4461-4473.

Gurvich, A. S., September, 1961, A Measurement of the Asymmetry Coefficient of the Velocity-Difference Distribution in the Ground Layer of the Atmosphere, Doklady Akademii Nauk SSSR, Vol. 134 (5), p. 1073-1075.

Hines, William W. and Douglas C. Montgomery, 1972, Probability and Statistics in Engineering and Management Science, The Ronald Press Company, New York, p. 509.

Jacobi, W., and K. Andre', July 1, 1963, The Vertical Distribution of Radon 222, Radon 220 and Their Decay Products in the Atmosphere, Journal of Geophysical Research, Vol. 68 (13), p. 3799-3814.

Karman, Theodore de, and Leslie Howarth, 1938, On the Statistical Theory of Isotropic Turbulence, Proceedings of the Royal Society of London, Series A, Vol. 164, p. 192-215.

Karol, I. L., 1966, Quantitative Investigation of Stratospheric Mixing Processes by Means of Long Lived Radon Decay Products, Tellus, Vol. 18 (2), p. 337-344.

Kolmogrov, A. N., 1941a, The Local Structure of Turbulence in Incompressible Viscous Fluid for Very Large Reynolds Numbers, Comptes Rendus (Doklady) de l'Academie des Sciences de l'URSS, Vol. 30 (4), p. 301-305.

__________, 1941b, Dissipation of Energy in the Locally Isotropic Turbulence, Comptes Rendus (Doklady) de l'Academie des Sciences de l'URSS, Vol. 32 (1), p. 16-18.

Lilly, D. K., D. E. Waco, and S. I. Adelfang, June, 1974, Stratospheric Mixing Estimated from High-Altitude Turbulence Measurements, Journal of Applied Meteorology, Vol. 13 (4), p. 488-493.

Lumley, John L. and Hans A. Panofsky, 1964, The Structure of Atmospheric Turbulence, Interscience Publishers, John Wiley & Sons, New York, p. 239.

Obukhov, A. M., 1951, Investigations of the Microstructure of the Wind in the Atmosphere Near the Surface, Izvestiya Akademii Nauk SSSR, Geophys Series 1951, No. 3, p. 49.

Panofsky, Hans A. and F. Pasquill, October, 1963, The Constant in the Kolmogrov Law, Quarterly Journal of the Royal Meteorological Society, Vol. 89 (382), p. 550-551.

Pasquill, F., 1962, Atmospheric Diffusion, D. Van Nostrand Company Ltd., London, p. 297.

Pond, S., R. W. Stewart, and R. W. Burling, July, 1963, Turbulence Spectra in the Wind Over Waves, Journal of the Atmospheric Sciences, Vol. 20, p. 319-324.

Readings, C. J. and D. R. Rayment, December, 1969, The High-Frequency Fluctuation of the Wind in the First Kilometer of the Atmosphere, Radio Science, Vol. 4 (12), p. 1127-1131.

Reed, Richard J. and Kenneth E. German, May, 1965, A Contribution to the Problem of Stratospheric Diffusion by Large-Scale Mixing, Monthly Weather Review, Vol. 93 (5), p. 313-321.

Reiter, Elmar R., December, 1969, Structure of Vertical Wind Profiles, Radio Science, Vol. 4 (12), p. 1133-1136.

Reynolds, Osborne, 1895, On the Dynamical Theory of Incompressible Fluids and the Determination of the Criterion, Philosophical Transactions of the Royal Society of London, Series A, Vol. 186, part 1, p. 123-164.

Richardson, Lewis F., July 1920, The Supply of Energy From and To Atmospheric Eddies, Proceedings of the Royal Society of London, Series A, Vol. 97 (A686), p. 354-373.

__________, 1952, Reply to W. C. Swinbank, Quarterly Journal of the Royal Meteorological Society, Vol. 78, p. 422-425.

Schlichting, H., 1933, Nachr. Ges Wiss. Gottingen, Math-physik. Klasse, 2, 181.

Stewart, H. J., 1945, Kinematics and Dynamics of Fluid Flow, Handbook of Meteorology, p. 412-500, F. A. Berry, Jr., E. Bollay, and Norman R. Beers (Eds), McGraw-Hill Book Company, New York.

Stewart, R. W., 1951, Triple Velocity Correlations in Isotropic Turbulence, Proceedings of the Cambridge Philosophical Society, Vol. 47 (2), p. 146-157.

Sutton, O. G., 1953, Micrometeorology, McGraw-Hill Book Company, New York, p. 333.

Taylor, G. I., 1915, Eddy Motion in the Atmosphere, Philosophical Transactions of the Royal Society of London, Series A, Vol. 215, p. 1-26.

_________, 1921, Diffusion by Continuous Movements, Proceedings of the London Mathematical Society, Vol. 2 (20), p. 196-202.

_________, 1935, Statistical Theory of Turbulence, pts. 1-4, Proceedings of the Royal Society of London, Series A, Vol. 151, p. 421-478.

_________, 1938, The Spectrum of Turbulence, Proceedings of the Royal Society of London, Series A, Vol. 164 (A919), p. 453-465.

Taylor, R. J., April, 1952, The Dissipation of Kinetic Energy in the Lowest Layer of the Atmosphere, Quarterly Journal of the Royal Meteorological Society, Vol. 78 (336), p. 179-185.

Tennekes, H. and John L. Lumley, 1972, A First Course in Turbulence, The MIT Press, Cambridge, Mass., p. 300.

Tollmien, W., 1929, Nachr, Ges. Wiss. Gottingen, Math-physik, Klasse, p. 21.

Townsend, A. A., 1948, Experimental Evidence for the Theory of Local Isotrophy, Proceedings of the Cambridge Philosophical Society, Vol. 44 (4), p. 560-565.

U. S. Environmental Science Services Administration, National Aeronautics and Space Administration, U. S. Air Force, 1966, U. S. Standard and Atmosphere Supplements, 1966, Government Printing Office, Washington, D. C.

Vinnichenko, Nikolay K. and John A. Dutton, December, 1969, Empirical Studies of Atmospheric Structure and Spectra in the Free Atmosphere, Radio Science, Vol. 4 (12), p. 1115-1126.

Wofsy, Steven C. and Michael B. McElroy, May 20, 1973, On Vertical Mixing in the Upper Stratosphere and Lower Mesophere, Journal of Geophysical Research, Vol. 78 (15), p. 2619-2624.

Zubkovski, S. L., 1962, Frequency Spectra Pulsations of the Horizontal Component of Wind Velocity in the Surface Air Layer, Investiya Akademii Nauk SSSR, Geophys Series 1962, p. 1425-1433.